

DAY — **09**

SEAT NUMBER

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2025 II 22

1100

**J-312**

(E)

**MATHEMATICS & STATISTICS (40)**  
**(ARTS & SCIENCE)**

Time : 3 Hrs.

( 8 Pages )

Max. Marks : 80

**General instructions :**

*The question paper is divided into **FOUR** sections.*

(1) **Section A :** *Q. 1 contains **Eight** multiple choice type questions carrying **Two** marks each.*

*Q. 2 contains **Four** very short answer type questions carrying **One** mark each.*

(2) **Section B :** *This section contains **Twelve** short answer type questions carrying **Two** marks each.*

*(Attempt any **Eight**)*

(3) **Section C :** *This section contains **Twelve** short answer type questions carrying **Three** marks each.*

*(Attempt any **Eight**)*

(4) **Section D :** *This section contains **Eight** long answer type questions carrying **Four** marks each.*

*(Attempt any **Five**)*

(5) *Use of log table is allowed. Use of calculator is not allowed.*

(6) *Figures to the right indicate full marks.*

0 3 1 2

- (7) Use of graph paper is not necessary. Only rough sketch of graph is expected.
- (8) For each multiple choice type of questions, only the first attempt will be considered for evaluation.
- (9) Start answer to each section on a new page.

## SECTION – A

**Q. 1. Select and write the correct answer of the following multiple choice type of questions : [16]**

- (i) If  $A = \{1, 2, 3, 4, 5\}$  then which of the following is not true ?
- (a)  $\exists x \in A$  such that  $x + 3 = 8$
- (b)  $\exists x \in A$  such that  $x + 2 < 9$
- (c)  $\forall x \in A, x + 6 \geq 9$
- (d)  $\exists x \in A$  such that  $x + 6 < 10$  (2)
- (ii) In  $\triangle ABC$ ,  $(a + b) \cdot \cos C + (b + c) \cos A + (c + a) \cdot \cos B$  is equal to \_\_\_\_.
- (a)  $a - b + c$
- (b)  $a + b - c$
- (c)  $a + b + c$
- (d)  $a - b - c$  (2)
- (iii) If  $|\vec{a}| = 5$ ,  $|\vec{b}| = 13$  and  $|\vec{a} \times \vec{b}| = 25$  then  $|\vec{a} \cdot \vec{b}|$  is equal to \_\_\_\_.
- (a) 30 (b) 60
- (c) 40 (d) 45 (2)

- (iv) The vector equation of the line passing through the point having position vector  $4\hat{i} - \hat{j} + 2\hat{k}$  and parallel to vector  $-2\hat{i} - \hat{j} + \hat{k}$  is given by \_\_\_\_\_.  
 (a)  $(4\hat{i} - \hat{j} - 2\hat{k}) + \lambda(-2\hat{i} - \hat{j} + \hat{k})$   
 (b)  $(4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$   
 (c)  $(4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-2\hat{i} - \hat{j} - \hat{k})$   
 (d)  $(4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-2\hat{i} - \hat{j} + \hat{k})$  (2)
- (v) Let  $f(1) = 3$ ,  $f'(1) = -\frac{1}{3}$ ,  $g(1) = -4$  and  $g'(1) = -\frac{8}{3}$ . The derivative of  $\sqrt{[f(x)]^2 + [g(x)]^2}$  w.r.t.  $x$  at  $x = 1$  is \_\_\_\_\_.  
 (a)  $-\frac{29}{25}$  (b)  $\frac{7}{3}$   
 (c)  $\frac{31}{15}$  (d)  $\frac{29}{15}$  (2)
- (vi) If the mean and variance of a binomial distribution are 18 and 12 respectively, then  $n$  is equal to \_\_\_\_\_.  
 (a) 36 (b) 54  
 (c) 18 (d) 27 (2)
- (vii) The value of  $\int x^x (1 + \log x) dx$  is equal to \_\_\_\_\_.  
 (a)  $\frac{1}{2}(1 + \log x)^2 + c$  (b)  $x^{2x} + c$   
 (c)  $x^x \cdot \log x + c$  (d)  $x^x + c$  (2)
- (viii) The area bounded by the line  $y = x$ , X-axis and the lines  $x = -1$  and  $x = 4$  is equal to \_\_\_\_\_.  
 (in square units)  
 (a)  $\frac{2}{17}$  (b) 8  
 (c)  $\frac{17}{2}$  (d)  $\frac{1}{2}$  (2)

**Q. 2.**      **Answer the following questions :**      **[4]**

(i)      Write the negation of the statement : ' $\exists n \in N$  such that  $n+8 > 11$ '      (1)

(ii)      Write unit vector in the opposite direction to  $\vec{u} = 8\hat{i} + 3\hat{j} - \hat{k}$ .      (1)

(iii)      Write the order of the differential equation 
$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$$
      (1)

(iv)      Write the condition for the function  $f(x)$ , to be strictly increasing, for all  $x \in R$ .      (1)

## SECTION – B

**Attempt any EIGHT of the following questions :**      **[16]**

**Q. 3.**      Using truth table, prove that the statement patterns  $p \leftrightarrow q$  and  $(p \wedge q) \vee (\sim p \wedge \sim q)$  are logically equivalent.      (2)

**Q. 4.**      Find the adjoint of the matrix  $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ .      (2)

**Q. 5.**      Find the general solution of  $\tan^2 \theta = 1$ .      (2)

**Q. 6.**      Find the co-ordinates of the points of intersection of the lines represented by  $x^2 - y^2 - 2x + 1 = 0$ .      (2)

**Q. 7.**      A line makes angles of measure  $45^\circ$  and  $60^\circ$  with the positive directions of the  $Y$  and  $Z$  axes respectively. Find the angle made by the line with the positive direction of the  $X$ -axis.      (2)

**Q. 8.** Find the vector equation of the plane passing through the point having position vector  $2\hat{i} + 3\hat{j} + 4\hat{k}$  and perpendicular to the vector  $2\hat{i} + \hat{j} - 2\hat{k}$ . (2)

**Q. 9.** Divide the number 20 into two parts such that sum of their squares is minimum. (2)

**Q. 10.** Evaluate :  $\int x^9 \cdot \sec^2(x^{10}) dx$  (2)

**Q. 11.** Evaluate :  $\int \frac{1}{25 - 9x^2} dx$  (2)

**Q. 12.** Evaluate :  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 - \sin x} dx$  (2)

**Q. 13.** Find the area of the region bounded by the parabola  $y^2 = 16x$  and its latus rectum. (2)

**Q. 14.** Suppose that  $X$  is waiting time in minutes for a bus and its p.d.f. is given by :

$$f(x) = \frac{1}{5}, \text{ for } 0 \leq x \leq 5$$
$$= 0, \text{ otherwise.}$$

Find the probability that :

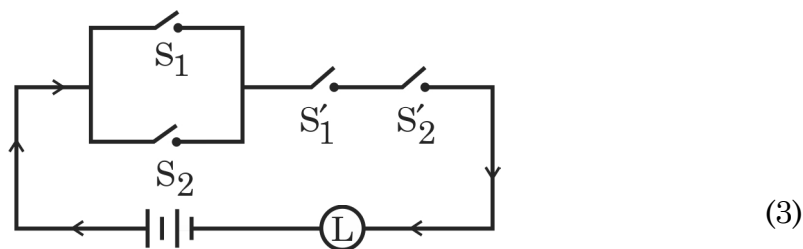
- (i) waiting time is between 1 to 3 minutes.
- (ii) waiting time is more than 4 minutes. (2)

## SECTION – C

Attempt any EIGHT of the following questions :

[24]

- Q. 15.** Express the following switching circuit in the symbolic form of logic. Construct the switching table and interpret it :



- Q. 16.** Prove that :  $2 \tan^{-1} \left( \frac{1}{3} \right) + \cos^{-1} \left( \frac{3}{5} \right) = \frac{\pi}{2}$ . (3)

- Q. 17.** In  $\triangle ABC$  if  $a = 13$ ,  $b = 14$ ,  $c = 15$  then find the values of  
(i)  $\sec A$  (ii)  $\operatorname{cosec} \frac{A}{2}$ . (3)

- Q. 18.** A line passes through the points  $(6, -7, -1)$  and  $(2, -3, 1)$ . Find the direction ratios and the direction cosines of the line. Show that the line does not pass through the origin. (3)

- Q. 19.** Find the cartesian and vector equations of the line passing through  $A(1, 2, 3)$  and having direction ratios 2, 3, 7. (3)

- Q. 20.** Find the vector equation of the plane passing through points  $A(1, 1, 2)$ ,  $B(0, 2, 3)$  and  $C(4, 5, 6)$ . (3)

- Q. 21.** Find the  $n^{\text{th}}$  order derivative of  $\log x$ . (3)

- Q. 22.** The displacement of a particle at time  $t$  is given by  $s = 2t^3 - 5t^2 + 4t - 3$ . Find the velocity and displacement at the time when the acceleration is  $14 \text{ ft/sec}^2$ . (3)

- Q. 23.** Find the equations of tangent and normal to the curve  $y = 2x^3 - x^2 + 2$  at point  $\left(\frac{1}{2}, 2\right)$ . (3)
- Q. 24.** Three coins are tossed simultaneously,  $X$  is the number of heads. Find the expected value and variance of  $X$ . (3)
- Q. 25.** Solve the differential equation :  $x \frac{dy}{dx} = x \cdot \tan\left(\frac{y}{x}\right) + y$ . (3)
- Q. 26.** Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability that :  
 (i) all the five cards are spades.  
 (ii) none is spade. (3)

## SECTION – D

**Attempt any FIVE of the following questions : [20]**

- Q. 27.** Find the inverse of  $\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$  by elementary row transformations. (4)
- Q. 28.** Prove that homogeneous equation of degree two in  $x$  and  $y$ ,  $ax^2 + 2hxy + by^2 = 0$  represents a pair of lines passing through the origin if  $h^2 - ab \geq 0$ . Hence show that equation  $x^2 + y^2 = 0$  does not represent a pair of lines. (4)
- Q. 29.** Let  $\vec{a}$  and  $\vec{b}$  be non-collinear vectors. If vector  $\vec{r}$  is coplanar with  $\vec{a}$  and  $\vec{b}$  then show that there exist unique scalars  $t_1$  and  $t_2$  such that  $\vec{r} = t_1 \vec{a} + t_2 \vec{b}$ . For  $\vec{r} = 2\hat{i} + 7\hat{j} + 9\hat{k}$ ,  $\vec{a} = \hat{i} + 2\hat{j}$ ,  $\vec{b} = \hat{j} + 3\hat{k}$ , find  $t_1, t_2$ . (4)

**Q. 30.** Solve the linear programming problem graphically.

Maximize :  $z = 3x + 5y$

Subject to :  $x + 4y \leq 24,$

$$3x + y \leq 21,$$

$$x + y \leq 9,$$

$$x \geq 0, y \geq 0$$

Also find the maximum value of  $z$ . (4)

**Q. 31.** If  $x = f(t)$  and  $y = g(t)$  are differentiable functions of  $t$  so that  $y$  is a function of  $x$  and if  $\frac{dx}{dt} \neq 0$

then prove that  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ .

Hence find the derivative of  $7^x$  w.r.t.  $x^7$ . (4)

**Q. 32.** Evaluate :  $\int e^{\sin^{-1} x} \left( \frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}} \right) dx$  (4)

**Q. 33.** Prove that :  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

Hence evaluate :  $\int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}} dx$  (4)

**Q. 34.** If a body cools from  $80^\circ\text{C}$  to  $50^\circ\text{C}$  at room temperature of  $25^\circ\text{C}$  in 30 minutes, find the temperature of the body after 1 hour. (4)



