MHT-CET 2024 Question Paper - Maths

4^{th} May 2024 (Shift – I)

1. Suppose A is any 3×3 non-singular matrix and $(A-3I)(A-5I)=0$ where $I=I_3$ and $O=O_3$. Here O_3 represent zero matrix of order 3 and I_3 is an identity matrix of order 3. If $\alpha A+\beta A^{-1}=4I$, then $\alpha+\beta$ is equal to
   (A) 13 (B) 7
   (C) 12 (D) 8

2. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if
   (A) $k=1$ or $k=-1$ (B) $k=0$ or $k=-3$
   (C) $k=3$ or $k=-3$ (D) $k=0$ or $k=3$

3. The variance of first 50 even natural numbers is
   (A) 833 (B) 473
   (C) $\frac{437}{4}$ (D) $\frac{833}{4}$

4. The statement pattern
   $[p\wedge(q\vee r)]\vee[\sim r\wedge\sim q\wedge p]$ is equivalent to
   (A) $q\vee r$ (B) $p\vee r$
   (C) q (D) p

5. If $8f(x)+6f\left(\frac{1}{x}\right)=x+5$ and $y=x^2f(x)$, then $\frac{dy}{dx}$ at $x=-1$ is
   (A) 14 (B) −14 (C) $\frac{1}{14}$ (D) $-\frac{1}{14}$

6. The number of ways in which 5 boys and 3 girls can be seated on a round table, if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is
   (A) 7! (B) $5\times6!$
   (C) $6\times6!$ (D) $5\times7!$

7. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is
   (A) $\frac{5}{17}$ (B) $\frac{6}{17}$ (C) $\frac{3}{17}$ (D) $\frac{4}{17}$

8. $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}\,dx$ equal to
   (A) $(x+1)e^{x+\frac{1}{x}}+c$, (where c is a constant of integration)
   (B) $-xe^{x+\frac{1}{x}}+c$, (where c is a constant of integration)
   (C) $(x-1)e^{x+\frac{1}{x}}+c$, (where c is a constant of integration)
   (D) $xe^{x+\frac{1}{x}}+c$, (where c is a constant of integration)

9. The function $f(x)=\frac{\log_e(\pi+x)}{\log_e(e+x)}$ is
   (A) increasing on $(0,\infty)$.
   (B) increasing on $\left(0,\frac{\pi}{e}\right)$, decreasing on $\left(\frac{\pi}{e},\infty\right)$.
   (C) decreasing on $(0,\infty)$.
   (D) decreasing on $\left(0,\frac{\pi}{e}\right)$, increasing on $\left(\frac{\pi}{e},\infty\right)$

10. Let $y=y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx}+y\cos x=4x,\ x\in(0,\pi)$.
    If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to
    (A) $-\frac{4}{9}\pi^2$ (B) $\frac{4}{9\sqrt{3}}\pi^2$
    (C) $\frac{-8}{9\sqrt{3}}\pi^2$ (D) $-\frac{8}{9}\pi^2$

11. The value of $I=\int\frac{x^2}{(a+bx)^2}dx$ is
    (A) $\frac{1}{b^3}\left[a+bx+2a\log|a+bx|-\frac{a^2}{a+bx}\right]+c$, (where c is the constant of integration)
    (B) $\frac{1}{b^3}\left[a+bx-2a\log|a+bx|+\frac{a^2}{a+bx}\right]+c$, (where c is the constant of integration)
    (C) $\frac{1}{b^3}\left[a+bx-2a\log|a+bx|-\frac{a^2}{a+bx}\right]+c$, (where c is the constant of integration)
    (D) $\frac{1}{b^3}\left[a+bx+2a\log|a+bx|+\frac{a^2}{a+bx}\right]+c$, (where c is the constant of integration)

12. Let $\bar{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}, \bar{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\bar{c} = \hat{i} + 2\hat{j} - 2\hat{k}$, where $\alpha, \beta \in \mathbb{R}$, be three vectors. If the projection of $\bar{a}$ on $\bar{c}$ is $\frac{10}{3}$ and $\bar{b} \times \bar{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $2\alpha + \beta$ is
(A) 3 (B) 4
(C) 5 (D) 6

13. If $4ab = 3h^2$, then the ratio of the slope of lines represented by $ax^2 + 2hxy + by^2 = 0$ is
(A) $\sqrt{3} : 1$ (B) $1 : \sqrt{3}$
(C) 1 : 3 (D) 3 : 1

14. If $\sin(\cot^{-1}(x + 1)) = \cos(\tan^{-1}x)$ then considering positive square roots, x has the value _______.
(A) 0 (B) $\frac{9}{4}$
(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

15. A random variable has the following probability distribution

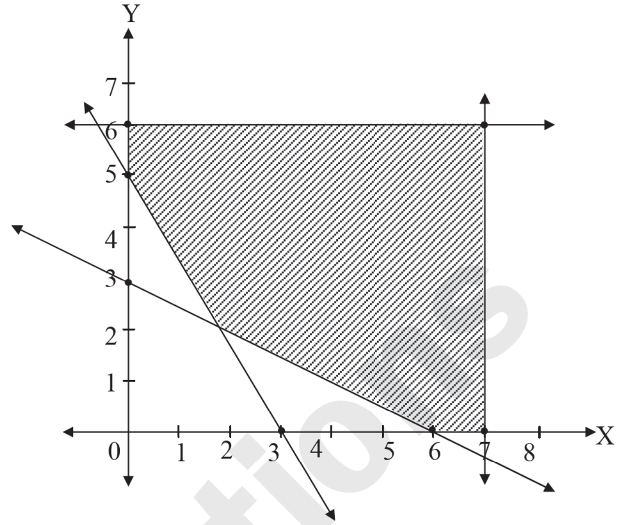
X:	0	1	2	3	4	5	6	7
P(x):	0	2p	2p	3p	p^2	$2p^2$	$7p^2$	2p

The then value of p is
(A) $\frac{1}{10}$ (B) $\frac{1}{30}$
(C) $\frac{1}{100}$ (D) $\frac{3}{20}$

16. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is
(A) 0.1 (B) 0.2
(C) 0.01 (D) 0.02

17. Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}, i = \sqrt{-1}$, where x and y are real numbers, then $(y - x)$ has the value
(A) –91 (B) –85
(C) 85 (D) 91

18. The shaded region in the following figure is the solution set of the inequations



(A) $x + 2y \le 6, 5x + 3y \ge 15, x \le 7, y \le 6, x, y \ge 0$
(B) $x + 2y \ge 6, 5x + 3y \ge 15, x \le 7, y \le 6, x, y \ge 0$
(C) $x + 2y \ge 6, 5x + 3y \le 15, x \ge 7, y \le 6, x, y \ge 0$
(D) $x + 2y \le 6, 5x + 3y \le 15, x \le 7, y \ge 6, x, y \ge 0$

19. Let $\bar{p}$ and $\bar{q}$ be the position vectors of P and Q respectively, with respect to O and $|\bar{p}| = p, |\bar{q}| = q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If OR and OS are perpendiculars, then
(A) $9p^2 = 4q^2$ (B) $4p^2 = 9q^2$
(C) $9p = 4q$ (D) $4p = 9q$

20. The value of a for which the volume of parallelepiped formed by $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum is
(A) $\frac{-1}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$
(C) $\sqrt{3}$ (D) $-\sqrt{3}$

21. The approximate value of $\log_{10} 1002$ is (Given $\log_{10} e = 0.4343$)
(A) 3.0117 (B) 3.0009
(C) 2.9999 (D) 3.1119

22. The sum of intercepts on coordinate axes made by tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is
(A) a (B) 2a
(C) $2\sqrt{a}$ (D) $\sqrt{2a}$

23. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is

(A) 1 : 6 (B) $\sqrt{3}:(2+\sqrt{3})$

(C) $1:(2+\sqrt{3})$ (D) 2 : 3

24. Considering only the Principal values of inverse functions, the set

$A=\left\{x\geq 0 \mid \tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}\right\}$

(A) contains two elements.
(B) contains more than two elements.
(C) is an empty set.
(D) is a singleton set.

25. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to line L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is _______ units.

(A) $\frac{23}{15}$ (B) $\sqrt{17}$

(C) $\frac{17}{\sqrt{15}}$ (D) $\frac{23}{\sqrt{17}}$

26. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left([x]+\log_e\left(\frac{1+x}{1-x}\right)\right)dx$, where $[x]$ represent greatest integer function, equals

(A) $-\frac{1}{2}$ (B) $\log_e\left(\frac{1}{2}\right)$

(C) $\frac{1}{2}$ (D) $2\log_e\left(\frac{1}{2}\right)$

27. If the function $f(x)=x^3+e^{\frac{x}{2}}$ and $g(x)=f^{-1}(x)$, then the value of $g'(1)$ is

(A) 1 (B) 0

(C) 2 (D) $\frac{1}{2}$

28. A wire of length 2 units is cut into two parts, which are bent respectively to form a square of side x units and a circle of radius of r units. If the sum of the areas of square and the circle so formed is minimum, then

(A) $2x=(\pi+4)r$ (B) $(4-\pi)x=\pi r$
(C) $x=2r$ (D) $2x=r$

29. Let $L_1:\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$ and

$L_2:\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$

be two given lines. Then the unit vector perpendicular to L_1 and L_2 is

(A) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(C) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i}-7\hat{j}-7\hat{k}}{\sqrt{99}}$

30. Let a, b $\in$ R. If the mirror image of the point p(a, 6, 9) w.r.t. line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, −a −9), then |a + b| is equal to

(A) 88 (B) 86
(C) 90 (D) 84

31. A random variable X has the following probability distribution

X:	1	2	3	4	5
P(X):	k^2	2k	k	2k	$5k^2$

Then $P(X > 2)$ is equal to

(A) $\frac{7}{12}$ (B) $\frac{23}{36}$

(C) $\frac{1}{36}$ (D) $\frac{1}{6}$

32. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is

(A) zero. (B) one.
(C) two. (D) three.

33. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right)$, $|x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

(A) $2f(x^2)$ (B) $-2f(x)$
(C) $(f(x))^2$ (D) $2f(x)$

34. Let the vectors $\bar{a},\bar{b},\bar{c}$ and $\bar{d}$ be such that $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=\bar{0}$. Let P_1 and P_2 be the planes determined by the pair of vectors $\bar{a},\bar{b}$ and $\bar{c},\bar{d}$ respectively, then the angle between P_1 and P_2 is

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

35. If $I=\int e^{\sin\theta}(\log\sin\theta+\operatorname{cosec}^2\theta)\cos\theta\ d\theta$, then I is equal to

(A) $e^{\sin\theta}(\log\sin\theta+\operatorname{cosec}^2\theta)+c$, (where c is a constant of integration)

(B) $e^{\sin\theta}(\log\sin\theta+\operatorname{cosec}\theta)+c$, (where c is a constant of integration)

(C) $e^{\sin\theta}(\log\sin\theta-\operatorname{cosec}\theta)+c$, (where c is a constant of integration)

(D) $e^{\sin\theta}(\log\sin\theta-\operatorname{cosec}^2\theta)+c$, (where c is a constant of integration)

36. The equation of the circle which has its centre at the point (3, 4) and touches the line $5x + 12y - 11 = 0$ is
(A) $x^2 + y^2 - 6x - 8y + 9 = 0$
(B) $x^2 + y^2 - 6x - 8y + 25 = 0$
(C) $x^2 + y^2 - 6x - 8y - 9 = 0$
(D) $x^2 + y^2 - 6x - 8y - 25 = 0$

37. A plane which is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, passes through (1, −2, 1). The distance of the plane from the point (1, 2, 2) is
(A) 0 units (B) 1 units
(C) $\sqrt{2}$ units (D) $2\sqrt{2}$ units

38. The value of the expression $\sqrt{3}\text{cosec}20^\circ - \sec 20^\circ$ is equal to
(A) 2 (B) $\frac{2\sin 20^\circ}{\sin 40^\circ}$
(C) 4 (D) $4\frac{\sin 20^\circ}{\sin 40^\circ}$

39. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three vectors having magnitude 1, 1 and 2 respectively. If $\bar{a} \times (\bar{a} \times \bar{c}) + \bar{b} = \bar{0}$, then the acute angle between $\bar{a}$ and $\bar{c}$ is
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

40. The area bounded between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq. units, then the value of a is
(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{3}$

41. The integral $\int \sec^{\frac{2}{3}} x \cdot \text{cosec}^{\frac{4}{3}} x \, dx$ is equal to
(A) $3(\tan x)^{-\frac{1}{3}} + c$, (where c is the constant of integration)
(B) $-\frac{3}{4}(\tan x)^{-\frac{4}{3}} + c$, (where c is the constant of integration)
(C) $-3(\cot x)^{-\frac{1}{3}} + c$, (where c is the constant of integration)
(D) $-3(\tan x)^{-\frac{1}{3}} + c$, (where c is the constant of integration)

42. If sum of two numbers is 3, then the maximum value of the product of first number and square of the second number is
(A) 6 (B) 4
(C) 5 (D) 3

43. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is
(A) $x\log|y| = x - 1$
(B) $x\log|y| = -2(x-1)$
(C) $x\log|y| = 2(x-1)$
(D) $x^2\log|y| = -2(x-1)$

44. If $y = \left((x+1)(4x+1)(9x+1)\ldots(n^2x+1)\right)^2$, then $\frac{dy}{dx}$ at $x = 0$ is
(A) $\frac{n(n+1)(2n+1)}{4}$ (B) $\frac{n(n+1)(2n+1)}{6}$
(C) $\frac{n(n+1)(2n+1)}{2}$ (D) $\frac{n(n+1)(2n+1)}{3}$

45. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability, that a student will get 4 or more correct answers just by guessing, is
(A) $\frac{10}{3^5}$ (B) $\frac{17}{3^5}$
(C) $\frac{13}{3^5}$ (D) $\frac{11}{3^5}$

46. A wet substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the open air loses half its moisture during the first hour, then the time t, in which 99% of the moisture will be lost, is
(A) $\frac{2\log 10}{\log 2}$ (B) $\frac{\log 10}{\log 2}$
(C) $\frac{3\log 10}{\log 2}$ (D) $\frac{1}{2}\frac{\log 10}{\log 2}$

47. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ has the value
(A) 2 (B) $\frac{1}{2}$ (C) 4 (D) 3

48. Let $a, b \in (a \neq 0)$. If the function f is defined as

$$f(x) = \begin{cases} \frac{2x^2}{a} & , 0 \le x < 1 \\ a & , 1 \le x < \sqrt{2} \\ \frac{2b^2 - 4b}{x} & , \sqrt{2} \le x < \infty \end{cases}$$

is continuous in the interval $[0, \infty)$, then an ordered pair (a, b) is
(A) $(-\sqrt{2}, 1-\sqrt{3})$ (B) $(\sqrt{2}, -1+\sqrt{3})$
(C) $(\sqrt{2}, 1-\sqrt{3})$ (D) $(-\sqrt{2}, 1+\sqrt{3})$

49. If $(p\wedge\sim q)\wedge(p\wedge r)\rightarrow\sim p\vee q$ is false, then the truth values of p, q and r are respectively

(A) T, T, T (B) F, F, F
(C) T, F, T (D) F, T, F

50. In ΔABC, with usual notations, if $b = 3$, $c = 8$, $m\angle A = 60^\circ$, then the circumradius of the triangle is ______ units.

(A) $\frac{7}{3}$ (B) $\frac{7\sqrt{2}}{3}$
(C) $\frac{7}{\sqrt{3}}$ (D) $\frac{7\sqrt{3}}{2}$