

## JEE MAIN 30 JANUARY 2024 SHIFT 1 QUESTION PAPER

### MATHEMATICS

1.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$
2.  $9 \int_0^9 \left[ \sqrt{\frac{10x}{x+1}} \right] dx = ?$  (Here,  $[ \ ]$  represents the greatest integer function).
3. A line passes through  $(9,0)$ , making angle  $30^\circ$  with positive direction of x-axis. It is rotated by angle of  $15^\circ$  with respect to  $(9,0)$ . Find the equation of the new line.
4. Find the eccentricity of an ellipse whose length of the minor axis is equal to half of the length between foci.
5. Find the number of integral terms in the binomial expansion:  
 $(7^{1/2} + 11^{1/6})^{824}$
6. Find the value of the maximum area possible (in sq.units) of  $\triangle ABC$  with vertices  $A(0, 0)$ ,  $B(x, y)$  and  $C(-x, y)$  such that  $y = -2x^2 + 54x$ .
7. For a non-zero complex number  $z$  satisfying  $z^2 + i\bar{z} = 0$ , then value of  $|z|^2$  is?
8. Given  $x^2 - 70x + 1 = 0$  with positive integral roots  $\alpha$  and  $\beta$  where one of the root is less than 10, and of  $\frac{\lambda}{2}$  and  $\frac{\lambda}{3}$  are not integers, then find value of  $\frac{\sqrt{\alpha-1} + \sqrt{\beta-1}}{|\alpha-\beta|}$ .
9. If  $|a| = 1$ ,  $|b| = 4$ ,  $a \cdot b = 2$  and  $c = 2(a \times b) - 3b$ , then what is the angle between  $b$  and  $c$ ?
10. If the foot of the perpendicular from  $(1, 2, 3)$  to the line  $(x+1)/2 = (y-2)/5 = (z-4)/1$  is  $(\alpha, \beta, \gamma)$ , then find  $\alpha + \beta + \gamma$ .
11. If  $y = f(x)$  is the solution of differential equation  $(x^2 - 1)dy = ((x^3 + 1) + \sqrt{1-x^2}) dx$  and  $y(0) = 2$ , then find  $y(\frac{1}{2})$ .
12. In an arithmetic progression, if the sum of 20 terms is 790 and the sum of 10 terms is 145, then  $S_{15} - S_5 = ?$
13. Set  $A = \{1, 2, 3, 4, 5, 6, 7\}$   
If the number of functions from Set A to Power Set A can be expressed as  $m^n$  (where  $m$  is the least integer), the find  $m + n$ .

14. In a class, there are 40 students.  
 16 students passed in Chemistry.  
 20 students passed in Physics.  
 25 students passed in Maths.  
 15 students passed in both Math and Physics.  
 15 students passed in both Math and Chemistry  
 10 students passed in both Physics and Chemistry.  
 Find the maximum number of students that passed in all the subjects.
15. Set  $S = \{0, 1, 2, 3, \dots, 10\}$   
 If a random ordered pair  $(x, y)$  of elements of  $S$  is chosen, then find probability that  $|x - y| > 5$ .
16. The domain of  $y = \cos^{-1} \left( \frac{2 - |x|}{4} \right) + \log(3 - x)^{-1}$  is  $[\alpha, \beta] - \{\gamma\}$ , then find  $\alpha + \beta + \gamma$ .
17. What is the range of  $r$  for which circles  $(x + 1)^2 + (y + 2)^2 = r^2$  and  $x^2 + y^2 - 4x - 4y + 4 = 0$  coincide at two distinct points.
18. Find the value of  $20M$ , if  $M$  is the median of the following data:

$x_i$	$f_i$	C.F.
0 - 4	2	2
4 - 8	4	6
8 - 12	7	13
12 - 16	8	21
16 - 20	6	27