JEE MAIN 27 JANUARY 2024 SHIFT 1 QUESTION PAPER

MATHEMATICS

1. $f(x) = \begin{cases} 2^{\frac{\sin(x-3)}{x-[x]}} & , \ x < 3 \\ \frac{a|-x^2-12+7|}{b(x^2+12x+7)} & , \ x > 3 \\ b & , \ x = 3 \end{cases}$

   If f(x) is continuous at x = 3, then (a, b) = ?

2. If $f(x) = \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   Statement I $\Rightarrow f(x) \cdot f(y) = f(x + y)$

   Statement II $\Rightarrow f(-x) = 0$ is invertible

3. $a = \lim_{x \to \infty} \left(\frac{\sqrt{1+\sqrt{1+x^2}}-\sqrt{2}}{x^4} \right)$ and $b = \lim_{x \to \infty} \left(\frac{sin^2x}{\sqrt{2}-\sqrt{1+cosx}} \right)$, then find ab^3.

4. $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx = a + b\sqrt{2} + c\sqrt{3}$, then $2a - 3b - 4c$ = ?

5. If $f(x) - f(y) = \ln\left(\frac{x}{y}\right) + x - y$, then find $\sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)$

6. A = {1,2,3,...,10}, S be the set of subsets of A and R = {(a, b): a, b ∈ S and a ∩ b ≠ Ø}. Then R is

7. Find p if:
   $3 + (3 + p)/4 + (3 + 2p)/4^2 + ...\infty = 8$

8. If a = i + 2j + k, b = 3 (i - j + k), a • c (scalar product) = 3 and a x c (vector product) = b, then find
   a • ((c x b) - b - c).

9. If a line L = 4x + 5y = 20 trisects two other lines L_1 and L_2 that pass through the origin, then find the tangent made by the line L.

10. If cos 2x - a sin x = 2a - 7, then range of a is?

11. If f(x)= x^3 + 2x^2*f'(1) + x*f''(2) + f'''(3). The value of f'(10) is equal to?

12. If $^{n-1}C_r = (k^2 - 8)^nC_{r+1}$, then find k.

13. If S = { z : | z + i | = | z - i | = | z - 1 |, z ∈ C}, then the number of elements in Set S = ?

14. If α is a root of $x^2 + x + 1 = 0$ satisfying $(1 + \alpha)^7 = a + b\alpha + c\alpha^2$, then the order triplet (a, b, c) is:

15. $S_1 = 3, 9, 15, \ldots$ 25 terms and $S_2 = 3, 8, 13, \ldots$ 37 terms, then the number of common terms in S_1, S_2 is equal to?

16. Shortest distance between the parabola $y^2 = 4x$ and $x^2 + y^2 - 4x - 16y + 64 = 0$ is equal to?

17. The shortest distance between the lines is?
    $(x - 1)/2 = (y + 1)/4 = (z - 1)/3$ and $(2x - 1)/5 = (y - 2)/3 = z/3$

18. The value of k for $(2k, 3k)$, $(0, 0)$, $(1, 0)$ and $(0, 1)$ to be on the circle is:

19. The vertices of a triangle ABC are A(1,2), B(- 3, 4) and C(5,8), then the orthocentre of $\triangle$ABC is?

20. $x^2/25 + y^2/16 = 1$ is the given ellipse. Find the length of the chord whose midpoint is $(1/2, 2/5)$.