# PART : MATHEMATICS

**1.** The value of integral $\int_0^{\pi/4} \frac{x\,dx}{\cos^4 2x + \sin^4 2x}$ is

(1) $\frac{\pi}{16}$ (2) $\frac{3\pi}{8}$ (3) $\frac{\pi^2}{16\sqrt{2}}$ (4) $\frac{\sqrt{3}\pi}{8}$

**Ans.** (3)

**Sol.** By property P– 6

$$\int_0^{\pi/8}\left(\frac{x}{\cos^4 2x+\sin^4 2x}+\frac{\frac{\pi}{4}-x}{\sin^4 2x+\cos^4 2x}\right)dx$$

$$\frac{\pi}{4}\frac{1}{2}\int_0^{\pi/4}\frac{d\theta}{\sin^4\theta+\cos^4\theta}=\frac{\pi}{8}\int_0^1\frac{1+t^2}{t^4+1}dt\,,$$

$t = \tan\theta$

$$=\frac{\pi}{8}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1=\frac{\pi^2}{16\sqrt{2}}$$

**2.** In sequences 3,7,11, ........, 404 and 4, 7, 10, ......, 403, the number of common terms are

**Ans.** (34)

**Sol.** 3, 7, 11, ......., ,404 C.D. = 4

4, 7, 10, ........, 403 C.D. = 3

LCM {4,3} = 12

7, 19, 31, ...... is sequence of common terms

$t_n = 7 + (n-1)12 \le 403$

$12n \le 408$

$n \le 34$

**3.** If 3, a, b, c are in AP and 3, a – 1, b + 1, c + 9 are in GP then AM of a, b, c is

**Ans.** (11)

**Sol.** $2a = b + 3$ ...........(1)

$2b = a + c$ ........... (2)

$\frac{a-1}{3}=\frac{b+1}{a-1}=\frac{c+9}{b+1}$ ........... (3)

$$\frac{\frac{b+3}{2}-1}{3}=\frac{b+1}{\frac{b+3}{2}-1}=\frac{2b-a+9}{b+1}$$

$$\frac{b+1}{6}=\frac{(b+1)\times 2}{b+1}=\frac{2b-\frac{b+3}{2}+9}{b+1}$$

$$\frac{b+1}{6}=2 \qquad\qquad 2b+2=\frac{4b-b-3+18}{2}$$

$$4b+4=3b+15$$

$$b=11 \qquad\qquad b=11$$

Now, a = 7, c = 22 – 7 = 15

Now A. M of a, b, c

$$=\frac{a+b+c}{3}=\frac{7+11+15}{3}=11$$

**4.** If $A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $C = ABA^T$, $X = A^TC^2A$, then det (X) is equal to

(1) 729 (2) 726 (3) 728 (4) 723

**Ans.** (1)

**Sol.** $x = A^TABA^TABA^TA$

$= (3I)\, I\, (3I)\, I\, (3I) = 27I \Rightarrow |X| = (27)^2 = 729$

**5.** The area bounded by $xy + 4y = 16$ and $x + y = 6$ is

(1) $32 - 42\ell n3$ (2) $42 - 32\ell n2$ (3) $30 - 32\ell n2$ (4) $33 + 16\ell n3$

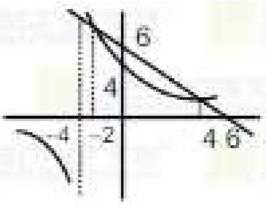
**Ans.** (3)

**Sol.** $(x + 4)\, y = 16$

$\Rightarrow (x + 4)(6 - x) = 16$

$6x - x^2 + 24 - 4x - 16 = 0$

$x^2 - 2x - 8 = 0$ ⟨ $-2$, $4$




$$\text{Area}=\int_{-2}^{4}\left((6-x)-\left(\frac{16}{x+4}\right)\right)dx = =6(6)-\frac{1}{2}(16-4)-16(\ell n(8)-\ell n2)=30-32\,\ell n2$$

6. The eccentricity of hyperbola $x^2 - y^2 \operatorname{cosec}^2\theta = 5$ is $\sqrt{7}$ times of eccentricity of ellipse $x^2 + y^2 \operatorname{cosec}^2\theta = 5$ then $\theta$ is where $0 < \theta < \frac{\pi}{2}$

(1) $\theta = \frac{\pi}{3}$ (2) $\theta = \frac{\pi}{2}$ (3) $\theta = \frac{\pi}{-3}$ (4) $\theta = \frac{2\pi}{3}$

**Ans.** (1)

**Sol.** Let $e_1$ eccentricity of ellipse and $e_2$ is eccentricity of hyperbola

$e_1 = \sqrt{1-\sin^2\theta} = \cos\theta$

$e_2 = \sqrt{1+\sin^2\theta}$

$e_2$ is $\sqrt{7}$ times of $e_1$

$\sqrt{1+\sin^2\theta} = \sqrt{7}\cos\theta$

$1 + \sin^2\theta = 7\cos^2\theta$

$2 = 8\cos^2\theta$

$\cos^2\theta = \frac{1}{4}$, $\cos\theta = \frac{1}{2}$, $\cos\theta = \frac{-1}{2}$ (rejected )

$\theta = \frac{\pi}{3}$

7. $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = \left[\left[(\vec{a} \times \vec{b}) \times \hat{j}\right] \times \hat{j}\right] \times \hat{j}$ then $\vec{c}.\left(-\hat{i} + \hat{j} + \hat{k}\right) = ?$

**Ans.** (9)

**Sol.** $\vec{c} = \left(\left((\vec{a}.\hat{j})\vec{b} - (\vec{b}.\hat{j})\vec{a}\right) \times \hat{j}\right) \times \hat{j}$

$= \left((\vec{b} - 2\vec{a}) \times \hat{j}\right) \times \hat{j}$

$= \left((\vec{b} - 2\vec{a}).\hat{j}\right)\hat{j} - (\vec{b} - 2\vec{a})\,(\hat{j}.\hat{j})$

$= \left((11\hat{i} + 2\hat{k}).\hat{j}\right)\hat{j} - (\vec{b} - 2\vec{a})$

$= -(\vec{b} - 2\vec{a}) = 2\vec{a} - \vec{b} = -11\hat{i} - 2\hat{k}$

$\vec{c}.\left(-\hat{i} + \hat{j} + \hat{k}\right) = 11 - 2 = 9$

8. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)}dx = a\pi + b\ln\left(3 + 2\sqrt{2}\right)$, then a + b is equal to

**Ans.** (4)

**Sol.** Using even odd property

$$= \int_0^{\frac{\pi}{2}} \left( \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} + \frac{e^{\sin x} 8\sqrt{2}\cos x}{(e^{\sin x}+1)(1+\sin^4 x)} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+\sin^4 x)} dx \quad \text{Put } t = \sin x \; = 8\sqrt{2}\int_0^1 \frac{dt}{1+t^4} = 4\sqrt{2}\int_0^1 \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt - 4\sqrt{2}t\int_0^1 \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$$

$$= 4\sqrt{2}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1 - 4\sqrt{2}\frac{1}{2\sqrt{2}}\ell n\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|_0^1 = 2\pi + 2\ell n(3+2\sqrt{2})$$

a + b = 4

**9.** $(\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10$ find sum of values of x

(1) 0 (2) 3 (3) 5 (4) 2

**Ans.** **(1)**

**Sol.** $(\sqrt{3}+\sqrt{2})^x = t$

$t + \frac{1}{t} = 10 \Rightarrow t^2 - 10t + 1 = 0 \Rightarrow t = \frac{10 \pm \sqrt{96}}{2}$

$t = 5 \pm 2\sqrt{6} \;\Rightarrow (\sqrt{3}+\sqrt{2})^x = 5 + 2\sqrt{6} \Rightarrow x = \pm 2$

sum = 0

**10.** $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ If the shortest distance between the above two lines is 1 then sum of possible values of $\lambda$.

(1) 0 (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $-2\sqrt{3}$

**Ans.** **(2)**

**Sol.** $S.D = \left| \frac{1}{|\vec{b}_1 \times \vec{b}_2|} \begin{vmatrix} \lambda-3 & 1 & -1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} \right|$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \hat{i}(3) - \hat{j}(-3) + 3\hat{k}$

$= \left| \frac{3(\lambda-\sqrt{3})+3-3}{\sqrt{9+9+9}} \right| = 1 \quad = \left| \frac{\lambda-\sqrt{3}}{\sqrt{3}} \right| = 1 \Rightarrow \lambda - \sqrt{3} = \pm\sqrt{3} \Rightarrow \lambda = 0, \; = 2\sqrt{3}$

**11.** $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$ , $y(0) = 1$, the find $\left[\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right]^2 = ?$

(1) $\log\frac{4}{4+5e}$ (2) $\frac{2}{1+\sqrt{6}}$ (3) $\frac{3}{3-\sqrt{e}}$ (4) $\frac{1}{2-\sqrt{e}}$

**Ans.** (4)

**Sol.** put $x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

Now $\frac{dt}{dx} - 1 = 2xt^3 - xt - 1$

$\frac{dt}{dx} = 2xt^3 - xt$

$\frac{1}{t^3}\frac{dt}{dx} + \frac{x}{t^2} = 2x$

Put $\frac{1}{t^2} = u$

$\frac{-2}{t^3}\frac{dt}{dx} = \frac{du}{dx}$

$\frac{-1}{2}\frac{du}{dx} + xu = 2x$

$\frac{du}{dx} - 2xu = -4x$

I.F $= e^{-\int 2x dx} = e^{-x^2}$

Solve $u \cdot e^{-x^2} = \int e^{-x^2}.(-4x)dx$

$\frac{e^{-x^2}}{t^2} = \int e^{-x^2}(-4x)dx$

$-x^2 = z$

$-2xdx = dz$

$\frac{e^{-x^2}}{(x+y)^2} = \int 2e^z dz$

$\frac{e^{-x^2}}{(x+y)^2} = 2e^z + c$

$$\frac{e^{-x^2}}{(x+y)^2} = 2e^{-x^2} + c$$

$$\frac{1}{(x+y)^2} = 2 + ce^{x^2}$$

at $x = 0,\ y = 1$

$$\frac{1}{1} = 2 + c$$

$c = -1$

Now $(x + y)^2 = \dfrac{1}{2 - e^{x^2}}$

at $x = \dfrac{1}{\sqrt{2}} \Rightarrow \left(y + \dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1}{2 - e^{\frac{1}{2}}}$

$$\left(y\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right)^2 = \left(\frac{1}{2 - \sqrt{e}}\right)$$

**12.** Two circles $c_1 : x^2 + y^2 - 4x - 6y - 3 = 0$ and $c_2 : x^2 + y^2 + 2x - 14y + \lambda$ meet at two distinct points then find the value of $\lambda$.

(1) $-31 < \lambda < 40$ (2) $-31 < \lambda < 49$ (3) $(-20 < \lambda < 49)$ (4) $(-11 < \lambda < 49)$

**Ans.** **(2)**

**Sol.** $c_1 = (2,3)$ $r_1 = 4$

$C_2 = (-1, 7)$ $r_2 = \sqrt{50-\lambda}$

$c_1c_2 = 5$

$|r_1 - r_2| < c_1 c_2 < r_1 + r_2$

$\left|4 - \sqrt{50-\lambda}\right| < 5 < 4 + \sqrt{50-\lambda}$

$\Rightarrow 1 < \sqrt{50-\lambda} \Rightarrow \lambda < 49$

$\Rightarrow -5 < \sqrt{50-\lambda} - 4 < 5$

$\Rightarrow \sqrt{50-\lambda} < 9 \Rightarrow 50 - \lambda < 81$

$\Rightarrow -31 < \lambda$

$\Rightarrow -31 < \lambda < 49$ Ans.

**13.** Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute then

(1) 47 (2) 53 (3) 43 (4) 51

**Ans.** **(4)**

**Sol.** Since rooms are identical so we can distribute in following way

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| 1 way = 1 | 0 | 0 | 0 | 5 |
| $\frac{5!}{4!\,1!}$ ways = 5 | 0 | 0 | 1 | 4 |
| $\frac{5!}{2!\,3!}$ ways = 10 | 0 | 0 | 2 | 3 |
| $\frac{5!}{3!\,1!\,1!} \times \frac{1}{2!} = 10$ | 0 | 1 | 1 | 3 |
| $\frac{5!}{1!\,2!\,2!\,2!} = 15$ | 0 | 1 | 2 | 2 |
| $\frac{5!}{1!\,1!\,1!\,2!} \times \frac{1}{3!} = 10$ | 1 | 1 | 1 | 2 |

Total 51 ways

**14.** x = x (t) solution of $(t+1)dx = [2x + (t+1)^3]dt$ $x(0) = 2$ then $x(1) =$

(1) 6 (2) 8 (3) 12 (4) 10

**Ans.** **(3)**

**Sol.** $\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^2$

$$\text{I.F.} = e^{\int \frac{-2}{t+1}dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int 1.dt \Rightarrow \frac{x}{(t+1)^2} = t + c$$

At $t = 0$, $x = 2$

$c = 2$

$$\Rightarrow \frac{x}{(t+1)^2} = t + 2$$

$x(1) = 12$

**15.** Let S = {1, 2, 3, .....20} , $R_1$ = {(a, b) : a divides b}, $R_2$ = {(a, b) : a is integral multiple of b} and a, b $\in$ S, then n $(R_1 - R_2)$ = ?

**Ans.** (46)

**Sol.** $R_1$ = {(1, 1) (1, 2) .....(1, 20) (2, 2) (2, 4) ..... (2, 20)
(3, 3) (3, 6) (3, 9) (3, 12) (3, 15) (3, 18)
(4, 4) (4, 8) (4, 12) (4, 16) (4, 20)
(5, 5) (5, 10) (5, 15) (5, 20) (6, 6) (6, 12) (6, 18)
(7, 7) (7, 14)
(8, 8) (8, 16)
(9, 9) (9, 18)
(10, 10) (10, 20)
(11, 11)......... (20, 20)

$n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 + 2 + 10 = 66$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$

$= 66 - 20 = 46$

**16.** A bag contain 8 Ball, whose colour are either white or black ball, 4 balls are drawn at random without replacement and it was found that 2 ball are white and other 2 ball are black. Then probability that the bag contains equal number of white and black balls is.

(1) $\frac{1}{5}$ (2) $\frac{1}{7}$ (3) $\frac{2}{5}$ (4) $\frac{2}{7}$

**Ans.** (4)

**Sol.** n(s) = there are 5 possible sample space.

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1)P\left(\frac{E}{A_1}\right)}{P(A_1)P\left(\frac{E}{A_1}\right) + P(A_2)P\left(\frac{E}{A_2}\right) + \ldots\ldots}$$

$$P\left(\frac{4B4W}{2B2W}\right) = \frac{P(4B4W)\times P\left(\frac{2B2W}{4B4W}\right)}{P(4B4W)\times P\left(\frac{2B2W}{4B4W}\right) + P(3B5W)P\left(\frac{2B2W}{3B5W}\right) + \ldots..}$$

$$= \frac{\frac{1}{5}\times\frac{{}^4C_2\,{}^4C_2}{{}^8C_4}}{\frac{1}{5}\times\frac{{}^4C_2\times{}^4C_2}{{}^8C_4} + \frac{1}{5}\times\frac{{}^5C_2\,{}^3C_2}{{}^8C_4}\times 2 + \frac{1}{5}\times\frac{{}^6C_2\,{}^2C_2}{{}^8C_4}\times 2} = \frac{36}{36+60+30} = \frac{36}{126} = \frac{6}{21} = \frac{2}{7}$$

**17.** If $x + 2y + 3z = 81$, $x, y, z \in W$ find the number of solutions

**Ans.** (588)

**Sol.** $z = 0,\ x + 2y = 81 \Rightarrow$ 41 Solutions

$z = 1,\ x + 2y = 78 \Rightarrow$ 40 Solutions

$z = 2,\ x + 2y = 75 \Rightarrow$ 38 Solutions

$z = 27\ \ x + 2y = 0 \Rightarrow$ 1 Solutions

number of solutions = $(1 + ...... + 41) - (3 + ..... + 39)$

$= 41 \times 21 - 3\times \dfrac{13\times 14}{2} = 21 \times 28 = 588$

**18.** Given $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2$ and $y = 9f(x)x^2$. An interval on which y is strictly increasing.

(1) $\left(0,\frac{1}{\sqrt{5}}\right)$ (2) $\left(\frac{-1}{\sqrt{5}},0\right)$ (3) $\left(\frac{-1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$ (4) $\left(-\infty,\frac{-1}{\sqrt{5}}\right)$

**Ans.** (2)

**Sol.** $5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2$

$\Rightarrow 25f(x) + 20f\left(\frac{1}{x}\right) = 5x^2 - 10$

$\Rightarrow 16f(x) + 20f\left(\frac{1}{x}\right) = \frac{4}{x^2} - 8$

$\Rightarrow 9f(x) = 5x^2 - 10 - \frac{4}{x^2} + 8$

$y = 9f(x)\,x^2 = 5x^4 - 2x^2 - 4$

$\frac{dy}{dx} = 20x^3 - 4x = 4x\left(5x^2 - 1\right)$

in $\left(-\sqrt{\frac{1}{5}},0\right)$ increasing

**19.** $f : R \to R$ be defined by $f(x) = \begin{cases} \dfrac{a - b\cos 2x}{x^2} & ;\ x < 0 \\ x^2 + cx + 2 & ;\ 0 \le x \le 1 \\ 2x + 1 & ;\ x > 1 \end{cases}$ If f is continuous and M is the number of points where it is not differentiable then m + a + b + c

**Ans.** (2)

**Sol.** $f(0^-) = f(0) \Rightarrow 2b = 2 \Rightarrow b = 1$

$f(1) = f(1^+) \Rightarrow 3 + c = 3 \Rightarrow c = 0$

Now $f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2}, & x < 0 \\ x^2 + 2, & 0 \le x \le 1 \end{cases}$

$2x + 1 \,,\ 1 < x$

$\begin{cases} \dfrac{2\sin^2 x}{x^2}, & x < 0 \\ x^2 + 2, & 0 \le x \le 1 \end{cases}$

$2x + 1,\ 1 < x$

clearly differentiable everywhere so m = 0

$m + a + b + c = 2$

**20.** Let $f(x) = \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$. If $f(0^+) = R$, $f(0^-) = L$ then the value of $\dfrac{16}{\pi^2}(L^2 + R^2)$ is

**Ans.** (9)

**Sol.** $L = \lim\limits_{x \to 0^-} \dfrac{\cos^{-1}(1-(x+1)^2)\sin^{-1}(1-(1+x))}{(x+1)(1-(x+1)^2)}$, $\{x\} = x + 1$

$= \lim\limits_{x \to 0^-} \dfrac{\cos^{-1}(-x^2-2x)\sin^{-1}(-x)}{(x+1)(-x^2-2x)}$

$= \dfrac{\cos^{-1}(0)}{1}\dfrac{1}{2}(1) = \dfrac{\pi}{4}$

$R = \lim\limits_{x \to 0^+} \dfrac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x(1-x^2)}$, $\{x\} = x$

$\lim\limits_{x \to 0^-} \dfrac{\dfrac{-1(-2x)\frac{\pi}{2}}{2\sqrt{1-(1-x^2)^2}}}{1}$

$\lim\limits_{x \to 0^+} \dfrac{\sqrt{2-x^2}}{1}\dfrac{\pi}{2} = \dfrac{\pi}{\sqrt{2}}$

$\dfrac{16}{\pi^2}(L^2 + R^2) = \dfrac{16}{\pi^2}\left(\dfrac{\pi^2}{16} + \dfrac{\pi^2}{8}\right) = 3$